SIT-HEP/TM-46

Curvatons and inhomogeneous scenarios with deviation from slow-roll

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Abstract

The spectral index is studied at the point where scalar fields deviate from slow-roll during inflation. Considering the deviation that may cause a significant difference to the time derivative of the Hubble parameter and also to the terms in the evolution equation, we show how the deviation affects the spectral index of the curvature perturbations. Considering conventional inflation, curvatons and other inhomogeneous scenarios as mechanisms for generating the cosmological perturbation, we examine whether the spectral index induced by the deviation from the standard slow-roll can explain the spectral index $n-1>0$ at $k=0.002/\mathrm{Mpc}$ while keeping $n-1<0$ at a smaller scale.

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1 Introduction

While the traditional inflationary scenario based on the slow-roll approximation is considered to be broadly accurate [1], there may be some shift from the slow-roll trajectory [2] during inflation. Besides the possibility of oscillating inflation [3], deviation from the slow-roll may occur in the earliest stage of inflation or just after the inflaton experiences a gap in the single-field or hybrid-type potential.[2] Allowing a short period of deviation from the slow-roll, the most significant effect may occur when the inflaton stops during inflation, where the inflaton turns around. Expressing the curvature perturbation for slow-roll inflation as

$$\mathcal{R}_k \simeq -\frac{H}{\dot{\phi}}\delta\phi_k, \tag{1.1}$$

the divergence at the turnaround is brought about by $\dot{\phi}$ in the denominator. This issue has been studied by Seto et. al. in Ref. [4], using the standard formalism of cosmological perturbations. They found that Eq. (1.1) is not correct when inflation stops, but that the correct answer can be obtained by replacing $\dot{\phi}$ by the slow-roll velocity $\dot{\phi}_s \equiv -V_\phi/3H$, where V_ϕ is the derivative of the potential with respect to the inflaton. The correct answer is thus given by

$$\mathcal{R}_k \simeq -\frac{H}{\dot{\phi}_s}\delta\phi_k. \tag{1.2}$$

The form of the curvature perturbation is the same as that for conventional slow-roll inflation. The δN formalism is very useful for understanding this result. The number of e-foldings N is given by

$$N(t_e) \equiv \int_{t_N}^{t_e} H dt = \int_{\phi_N}^{\phi_e} \frac{H}{\dot{\phi}} d\phi, \tag{1.3}$$

where $t = t_N$ is the time when the perturbation crosses the horizon. Here, we have introduced $\phi_N \equiv \phi(t_N)$ and $\phi_e \equiv \phi(t_e)$ for simplicity. Following the model considered in Ref. [4], we split the inflaton velocity as

$$\dot{\phi} = \dot{\phi}_s + \dot{\phi}_d, \tag{1.4}$$

where $\dot{\phi}_s$ satisfies the slow-roll condition while $\dot{\phi}_d$ is the decaying velocity that follows $\dot{\phi}_d \propto e^{-3Ht}$. Note that the time elapsed during the decaying phase (where $\dot{\phi}_d \neq 0$ is

[2] The same deviation may appear for other light scalar fields such as curvatons and moduli fields. In Sec. 3, we consider similar situations for these alternatives.

significant) is determined by the Hubble parameter H and $\dot{\phi}_d$ at the horizon crossing.[3]. In terms of the δN formalism [5], the time passed after the horizon crossing is given by

$$\Delta t \equiv \Delta t_{dec} + \Delta t_{slow} \equiv (t_b - t_N) + (t_e - t_b), \tag{1.5}$$

where Δt_{dec} and Δt_{slow} are the time passed during the decaying phase and the slow-roll phase, which are shown in Fig.1. $\dot{\phi}_s$ is constant during the decaying phase, consistent with the usual assumption in slow-roll inflation. If the evolution equation of the inflaton field is homogeneous during the decaying phase, the fluctuation at the boundary $\delta\phi_b$ appears as a parallel displacement of $\delta\phi_N$.[4] As a result, the fluctuation of the time elapsed during inflation is given by $\delta(\Delta t) = \delta(\Delta t_{slow} + \Delta t_{dec}) \simeq \delta(\Delta t_{slow})$,[5] which suggests that the decaying phase does not play significant role in generating δN. Therefore, δN in this scenario must be calculated in the slow-roll phase, using the conventional slow-roll parameter of the potential at $\phi = \phi_b$. Here, the net fluctuation of the time $\delta(\Delta t)$ is not generated during the decaying phase. This result indicates that the curvature perturbation is determined by the slow-roll parameter of the potential

$$\epsilon_\phi \equiv \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2, \tag{1.6}$$

where M_p is the reduced Planck mass. Therefore, even if the inflaton stops ($\dot{\phi} = 0$) when it turns around at $\phi = \phi_N$, the curvature perturbation does not diverge since δN does not

[3]For simplicity, we do not consider the fluctuation of the inflaton velocity $\delta\dot{\phi}$. See Ref. [6] for more details.

[4]Of course, $\dot{\phi}_d$ does not cross $\dot{\phi}_d = 0$, but we can define the "boundary" at, for example, $|\dot{\phi}_d/\dot{\phi}_s| = 0.1$. In this case, the time elapsed between the turnaround and the boundary is calculated from $e^{-3H(\Delta t_{dec})} = 0.1$. It is then easy to calculate the width of the decaying phase. Considering the evolution equation for the inflaton field, the required conditions for the homogeneous evolution are: (1) homogeneous initial velocity, (2) homogeneous H and (3) homogeneous V. From the assumption that the inflaton velocity is homogeneous at $\phi = \phi_N$, we find homogeneous kinetic energy for the inflaton. However, there may be inhomogenities in V, which may be caused by the perturbation $\delta\phi_N$. Therefore, our argument is true for a sufficiently flat potential, while one must consider inhomogeneous evolution caused by δV and δV_ϕ if the inhomogenities in the evolution equation are significant. In this paper, we consider the former case following Ref. [4]. Note that our argument is not true for the model in which the inhomogenities created by $\delta\phi_N$ are not negligible in the evolution equation for the inflaton field.

[5] During the decaying phase, we find $\delta(\Delta N_{dec}) = \delta(H\Delta t_{dec}) = \delta H(\Delta t_{dec}) + H\delta(\Delta t_{dec})$. The inhomogenities related to δH is $\delta H(\Delta t_{dec}) \simeq \left[\frac{1}{6HM_p^2}\frac{dV}{d\phi}\delta\phi\right] \times \left[H^{-1}\right] \simeq \sqrt{\epsilon_\phi}H/M_p$, which can be neglected.

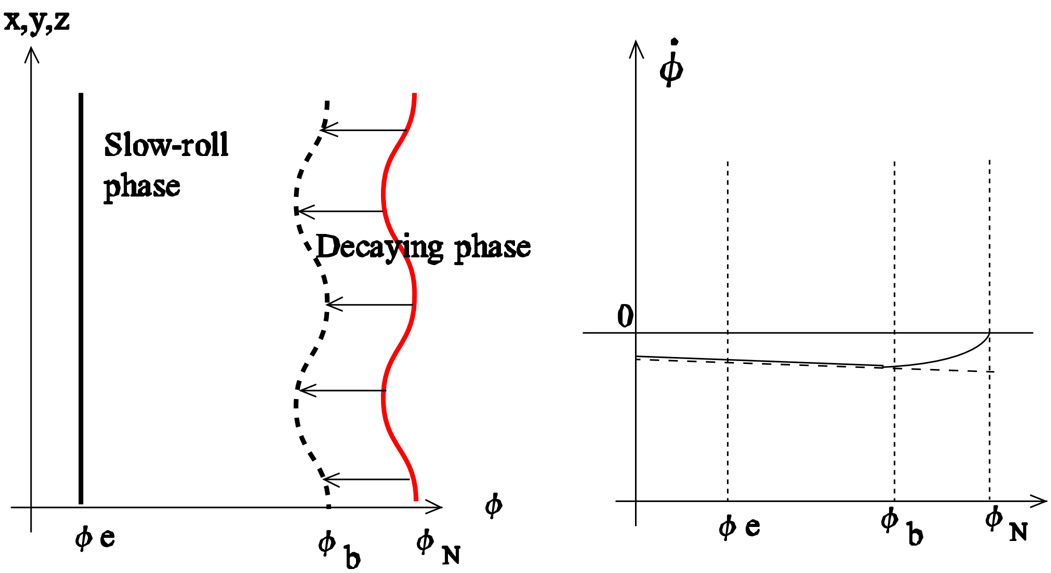

Figure 1: In the traditional inflationary scenario, $\delta N \simeq H\delta(\Delta t)$ is caused by the fluctuation of the distance between the start-line at ϕ_N (horizon crossing) and the goal-line at ϕ_e (end of inflation). In the right-hand image, we show the inflaton velocity (continuous line) with a small deviation from the slow-roll velocity (dotted line) during the decaying phase between $\phi = \phi_N$ and $\phi = \phi_b$.

depend on the inflaton velocity in the decaying phase. The result obtained by using the δN formalism is consistent with that obtained from the usual perturbation theory [4].

Although the final form of the curvature perturbation may look the same as conventional slow-roll inflation, we should remember that the Hubble parameter at the horizon crossing depends on the inflaton velocity. For a typical example, the difference is significant when we consider the time derivative of the Hubble parameter ($\dot{H} \propto \dot{\phi}^2$), which depends explicitly on the deviation of the inflaton velocity. Since the amplitude of the field fluctuation is determined by the Hubble parameter at the horizon crossing, the deviation from the slow-roll may lead to significant $\dot{\phi}_d$-dependence of the spectral index, since the spectral index measures the scale-dependence of the perturbation. This would also be true for many other scenarios such as curvatons [7, 8], inhomogeneous (p)reheating [9, 10, 11] or the hybrid of the two scenarios [12], as far as the amplitude of the seed fluctuations of light fields is determined by the Hubble parameter at the horizon crossing. Based on this

simple idea, we calculate the spectral index for the cosmological perturbations of various models, especially when there is a significant deviation from the slow-roll velocity.

1.1 Inflaton at a turnaround

In the standard inflation model, following the above argument, the primordial curvature perturbation $\mathcal{R}_k$ is equal to $-H\delta\phi_N/\dot{\phi}_s$ evaluated at $\phi = \phi_b$. This gives the spectrum of the curvature perturbation [1] as

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{\dot{\phi}_s}\right)^2 \left(\frac{H}{2\pi}\right)^2 = \frac{H^2}{8\pi^2 M_p^2 \epsilon_\phi} \tag{1.7}$$

where the slow-roll velocity is $\dot{\phi}_s \equiv -V_\phi/3H$. There is no significant difference in the form of the curvature perturbation, but the spectral index of the curvature perturbation given by

$$n - 1 \equiv \frac{d\ln \mathcal{P}_{\mathcal{R}}}{d\ln k} = -\frac{d\ln \epsilon_\phi}{d\ln k} + 2\frac{d\ln H}{Hdt} = -4\epsilon_\phi + 2\eta_\phi - 2\epsilon_H, \tag{1.8}$$

where the definitions of the slow-roll parameters are $\eta_\phi \equiv M_p^2 V_{\phi\phi}/V$ and $\epsilon_H \equiv -\dot{H}/H^2 = \dot{\phi}^2/2M_p^2 H^2$, should be different. Paying attention to the derivative[6]

$$\frac{d}{d\ln k} \simeq \frac{d}{Hdt} = \dot{\phi}\frac{d}{Hd\phi}, \tag{1.9}$$

the spectral index at a turnaround is

$$n - 1 \simeq 0. \tag{1.10}$$

This result suggests that $\dot{\phi} = 0$ during inflation may not explain $n - 1 > 0$ at $k = 0.002/\mathrm{Mpc}$ and $n - 1 < 0$ at $k = 0.05/\mathrm{Mpc}$ [13]. For example, the value of the spectral index [13] is

$$n_{0.002} = 1.21^{+0.13}_{-0.16} \tag{1.11}$$

at $k = 0.002/\mathrm{Mpc}$ and

$$n_{0.05} = 0.948^{+0.014}_{-0.018}, \tag{1.12}$$

which cannot be satisfied even if the inflaton stops during inflation.

[6] Here we use the relation $k = aH$, which leads to $d\ln k = \frac{da}{a} + \frac{dH}{H} = \left(H + \frac{\dot{H}}{H}\right) dt \simeq H\left(1 - \epsilon_\phi \frac{\dot{\phi}^2}{\dot{\phi}_s^2}\right) dt \simeq Hdt$.

Before the inflaton stops during inflation, the velocity of the inflaton may be much larger than the slow-roll velocity. Therefore, we will consider a more general situation with the inflaton velocity $\dot{\phi} \equiv -f_d \dot{\phi}_s$, $1/\sqrt{\epsilon_\phi} \gg 1 \geq f_d > 0$. Paying attention to the derivative

$$\frac{d}{d\ln k} = \frac{d}{H(1-\epsilon_\phi f_d^2)dt} \simeq -f_d \dot{\phi}_s \frac{d}{Hd\phi}, \tag{1.13}$$

the equation for the spectral index

$$n-1 \equiv \frac{d\ln \mathcal{P}_\mathcal{R}}{d\ln k} = -\frac{d\ln \epsilon_\phi}{d\ln k} + 2\frac{d\ln H}{Hdt} \tag{1.14}$$

leads to

$$n-1 \simeq -f_d\left(2\eta_\phi - 4\epsilon_\phi\right) - 2f_d^2\epsilon_\phi, \tag{1.15}$$

which may explain $n-1>0$ at $k=0.002/\mathrm{Mpc}$ for $f_d \leq 1$.

2 Curvatons and other alternatives

2.1 When inflaton deviates from slow-roll during inflation

Contrary to the result obtained for standard inflation, we find that in the curvaton model, a positive $n-1$ at $k=0.002/\mathrm{Mpc}$ is a natural consequence of $\dot{\phi}=0$. The spectral index for the curvaton model is given by [14]

$$n-1 \simeq 2\eta_\sigma - 2\epsilon_H. \tag{2.1}$$

Here, ϵ_σ is disregarded since it is typically very small compared to η_σ.[7] We assume ϵ_σ and η_σ are constants during inflation (i.e., the deviation occurs only for the inflaton field.). Then, the spectral index of the curvature perturbation that is related to the perturbation of the curvaton when the inflaton stops ($\dot{\phi}=0$) is

$$n-1 \simeq 2\eta_\sigma, \tag{2.2}$$

which may be positive at $k=0.002/\mathrm{Mpc}$,[8] while the spectral index at a smaller scale is

$$n-1 \simeq 2\eta_\sigma - 2\epsilon_\phi, \tag{2.3}$$

[7]For the curvaton velocity $\dot{\sigma} > \dot{\phi}$, we find $d\ln k \simeq H\left(1-\epsilon_\sigma\frac{\dot{\sigma}^2}{\dot{\sigma}_s^2}\right)dt \simeq Hdt$. The last approximation is true if $\dot{\sigma} \ll \dot{\sigma}_s/\sqrt{\epsilon_\sigma}$, which is consistent with the standard curvaton scenario that suggests very small ϵ_σ.

[8]It is possible to construct a curvaton model in which η_σ is negative during inflation [15]. We will not consider the possibility in this paper.

which must be negative. Note that in this model the flip in the sign of $n-1$ is not caused by the gap in the slow-roll parameters of the potential. The flip occurs because $\epsilon_H \propto \dot{\phi}^2$ disappears at the turnaround where the inflaton stops. For curvatons, the running spectral index in (1.11) and (1.12) leads to the condition $\epsilon_\phi > 0.04$, which is not constrained by the CMB normalization. Note that large ϵ_ϕ is favorable for the curvaton scenario, since a larger ϵ_ϕ decreases the inflaton-induced curvature perturbation. Therefore, the scenario of a running spectral index is more natural in the curvaton scenario.

Besides curvatons, as discussed above, inhomogeneous preheating is a possibile mechanism for the cosmological perturbation. The mechanism is consistent with the non-linear parameter that measures the non-Gaussianity of the spectrum. It also allows inflation with low-scale gravity [16]. Moreover, the mechanism can be used to generate the initial density perturbation of the curvatons [12]. Although the mechanisms for generating the curvature perturbation in inhomogeneous preheating scenarios are quite different from that in the curvaton model, there is no significant difference in the form of the spectral index [9]. The η-parameter of the light field is negative in a trapping inflation model [11], but in other scenarios a positive η can be taken. The spectral index in inhomogeneous preheating is given by Eq. (2.1), which naturally leads to $n-1>0$ when inflaton stops.

2.2 When curvatons deviate from slow-roll during inflation

As discussed in Section 1, deviation from the slow-roll may occur in the earliest stage of inflation or just after the field experiences a gap in the single-field or hybrid-type potential. Of course, the same deviation may appear for scalar fields other than the inflaton field. In this section, we will consider the spectral index when the curvaton has an initial velocity in the "wrong direction" (i.e., opposite to the potential gradient) in the earliest stage of inflation and then stops during inflation. For simplicity, we disregard deviations that may also appear for the inflaton field (i.e., we consider $\dot{\phi} = \dot{\phi}_s$ for the inflaton) and assume the deviation $\dot{\sigma} \neq \dot{\sigma}_s$ for the curvaton. The evolution of the fluctuation of the curvaton field is given in terms of the multi-field perturbation of the field equation [18]:

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + \left[\frac{k^2}{a^2} + m_\sigma^2 - \frac{1}{M_p^2 a^3}\frac{d}{dt}\left(\frac{a^3\dot{\sigma}^2}{H}\right)\right]\delta\sigma = \left[\frac{1}{M_p^2 a^3}\frac{d}{dt}\left(\frac{a^3\dot{\phi}\dot{\sigma}}{H}\right)\right]\delta\phi, \qquad (2.4)$$

where we considered a standard quadratic potential $V(\sigma) = \frac{1}{2}m_\sigma \sigma^2$ for the curvaton. We find on large scales that

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + \left[m_\sigma^2 - \frac{3\dot{\sigma}^2}{M_p^2} + \frac{\dot{\sigma}^2}{M_p^2}\frac{\dot{H}}{H^2} - \frac{2\ddot{\sigma}\dot{\sigma}}{M_p^2 H}\right]\delta\sigma + \left[-\frac{3\dot{\phi}\dot{\sigma}}{M_p^2} + \frac{\dot{\sigma}\dot{\phi}}{M_p^2}\frac{\dot{H}}{H^2} - \frac{\ddot{\sigma}\dot{\phi} + \ddot{\phi}\dot{\sigma}}{M_p^2 H}\right]\delta\phi = 0. \tag{2.5}$$

For an inflaton that satisfies $\dot{\phi} = \dot{\phi}_s$, the inflaton velocity $\dot{\phi}$ can be expressed in terms of the slow-roll parameter ϵ_ϕ:

$$\dot{\phi} = \dot{\phi}_s \equiv \sqrt{2\epsilon_\phi} H M_p. \tag{2.6}$$

Let us first consider what happens when the curvaton stops during inflation. Considering $\dot{\sigma} = 0$ and $\ddot{\sigma} = -V_\sigma$ and following the standard assumption $\epsilon_\sigma \ll \eta_\sigma$ for the curvaton potential, we find[9]

$$H^{-1}\dot{\delta\sigma} \simeq -\eta_\sigma \delta\sigma. \tag{2.7}$$

This equation gives the evolution of $\delta\sigma$ after the horizon crossing. In addition to the scale-dependence of the curvaton fluctuation $\delta\sigma$, which can be seen from the evolution equation, we must consider the variation in the initial value. Considering these effects, we find that the spectral index for the curvaton when the curvaton stops ($\dot{\sigma} = 0$) is given by

$$n - 1 = 2\eta_\sigma - 2\epsilon_H. \tag{2.8}$$

In this case, the running is very small since the slow-roll parameter ϵ_σ is typically very small.

Besides the possibility of $\dot{\sigma} = 0$, the running of the spectral index may be significant if the curvaton has a significant velocity $|\dot{\sigma}| \gg |\dot{\sigma}_s|$. Considering the deviation from the slow-roll, we split the curvaton velocity as

$$\dot{\sigma} = \dot{\sigma}_s + \dot{\sigma}_d \equiv f_d \dot{\sigma}_s, \tag{2.9}$$

where $\dot{\sigma}_s$ satisfies the slow-roll condition while $\dot{\sigma}_d$ is the decaying velocity that follows $\dot{\sigma}_d \propto e^{-3Ht}$. Substituting the curvaton velocity, we find from Eq. (2.4) that[10]

$$H^{-1}\dot{\delta\sigma} \simeq -\left(\eta_\sigma + 2f_d^2\epsilon_\sigma\right)\delta\sigma, \tag{2.10}$$

[9] Here we neglected the term related to $\ddot{\delta\sigma}$ as in the usual calculation, since for the terms in Eq. (2.5), there is no significant change in the approximation even if the curvaton stops during inflation. We neglected the term related to $\ddot{\sigma}\dot{\phi}\delta\phi$ assuming that $\sqrt{\epsilon_\sigma\epsilon_\phi} \ll \eta_\sigma$.

[10] The change in the slow-roll approximation may be significant if the deviation from the slow-roll is

where we have disregarded higher order terms and considered $\ddot{\sigma} \simeq -3H\dot{\sigma}$. The spectral index for the curvaton is thus given by

$$n - 1 = 4f_d^2\epsilon_\sigma + 2\eta_\sigma - 2\epsilon_H. \tag{2.11}$$

Here ϵ_H is given by $\epsilon_H \equiv \sum_i \dot{\phi}_i^{\,2}/2M_p^2H^2$, with the sum applying for all fields that have non-zero velocity during inflation. Note that the significant deviation ($f_d^2 > \epsilon_\phi/\epsilon_\sigma$) leads to $n - 1 > 0$.

Our conclusion in this section is that in the standard curvaton scenario $\dot{\sigma} = 0$ does not lead to a significant running of the spectral index, which explains $n - 1 > 0$ at $k = 0.002/\mathrm{Mpc}$, while the deviation $\dot{\sigma}^2 > (\epsilon_\phi/\epsilon_\sigma)\dot{\sigma}_s^2$ can lead to a spectral index of $n-1 > 0$ with significant running. Large ϵ_σ is required in order to explain the running of the spectral index with $\dot{\sigma} = 0$, which suggests that the curvaton scenario is not standard in this case.

3 Conclusions and discussions

We have studied the spectral index at a scale when inflaton deviates from slow-roll during inflation. It might be suspected that the curvature power spectrum would be considerably enhanced when the inflaton stops during inflaton. Following the discussions in Ref. [4], the formula for the spectrum of the density perturbation derived at $\dot{\phi} = 0$ seems to contain a solution singular at $\dot{\phi} = 0$. However, the solution is regular, as has been discussed in Ref. [19]. The enhancement may appear in a decaying mode of the curvature perturbation, but it does not appear in the nondecaying mode. Therefore, there is a significant enhancement only for the short-wavelength perturbations. The important result in Ref. [4] is that the final form of the curvature perturbation at large scales is the same as that obtained for conventional slow-roll inflation, even if the inflaton stops during inflation. The δN formalism is very useful for understanding why the deviation from the slow-roll does not alter the final form of the curvature perturbations at large scales.

Also of importance is the fact that although the final forms of the curvature perturbation look quite similar, there is still a significant difference from the standard inflation.

significant. However, as far as we are considering $|n-1| \ll 1$, we may use the usual approximation. We neglected the term related to $\ddot{\sigma}\dot{\phi}\delta\phi$, since the term is smaller than the term proportional to f_d^2.

For example, note that $\dot{H}$ is proportional to $\dot{\phi}^2$, which shows that the time-derivative of the Hubble parameter is significantly different from that for standard inflation. The differences lead to a crucial disparity in the spectral index. As a result, a significant difference appears in the time derivative of the seed-field perturbation, which causes a significant modification of the spectral index. Our results show that the modification is significant in the standard inflation model. Interestingly, the running of the spectral index is also significant in other alternatives such as curvatons and inhomogeneous scenarios.

In addition to the deviation $\dot{\phi} \neq \dot{\phi}_s$, we considered a similar situation for the curvaton field ($\dot{\sigma} \neq \dot{\sigma}_s$), which leads again to a significant running of the spectral index.

Despite the characteristic behavior of the scale-dependent spectrum that is expected in these models, the actual resolution of the current CMB data with respect to the scale dependence is not accurate enough to determine the origin of the spectrum in terms of the scale-dependence. If future observations provide more accurate data with respect to the scale dependence, we hope the signature of the initial state of the Universe may be detected in the deviation from the slow-roll.

4 Note added

The model discussed in Sec.2.1 may be similar to DBI-curvatons discussed in Ref.[17]. They considered two-field DBI action for the fields $\{\phi, \sigma\}$ and find that the sound velocity can be very small when $\dot{\phi} \gg \dot{\sigma}$ for the inflaton ϕ, which leads to some remarkable properties for the curvature perturbations caused by the curvaton field σ.

5 Acknowledgment

We wish to thank K.Shima for encouragement, and our colleagues at Tokyo University for their kind hospitality.

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